PHYSICAL REVIEW VOLUME 131, NUMBER 4 15 AUGUST 1963

Threshold Effects in n-d Scattering due to the Existence of the Di-Neutron†

R. Alzetta
Istituto di Fisica dell'Università di Trieste, and Istituto Nazionale di Fisica Nucleare, sottosezione di Trieste, Italy

AND

G. C. Ghirardi and A. Rimini
Istituto di Fisica dell'Università di Parma, and Istituto Nazionale di Fisica Nucleare, Gruppo di Parma, Italy

(Received 17 December 1962)

Use is made of a simple soluble model to evaluate the threshold effect which arises in the n-d scattering cross section if the di-neutron exists. The model contains four parameters. They permit a large set of experimental data to be reproduced remarkably well. Various binding energies of the hypothetical di-neutron are assumed. The effect is a rounded step. Its height is about 20% of the total cross section for any reasonable value of the binding energy, if the value 8.26 F is assumed for the n-d doublet scattering length.

INTRODUCTION

THE existence of a bound state of the system of two neutrons (di-neutron) is forbidden both by the hypothesis of charge independence and by that of charge symmetry of the nuclear forces. However, the accuracy with which these invariance principles are established experimentally is not such as to rule out slight differences between the p-p, n-p interactions and the n-n interaction, which would lead to a bound state of two neutrons corresponding to the virtual $^1S_0$ state of the deuteron. Some authors[1–11] have discussed this possibility in connection with the interpretation of several experiments; nevertheless, owing to the considerable uncertainties either experimental or interpretative, it can be said that the existence of such a bound di-neutron is, until now, neither proved nor disproved. Considering the importance that the detection of the di-neutron would have for the whole problem of nuclear forces, it is interesting to suggest direct experiments which could determine whether it exists.

As is well known, when in a scattering process the threshold energy of a two-body channel is attained, an anomaly comes out in the cross section consisting in the fact that its derivative is infinite.[12] Recently,[12,13] it has been suggested, in order to detect the di-neutron, to look for a threshold effect in the n-d elastic scattering due to the existence of the di-neutron. The possibility of using this method depends, of course, on the magnitude of the effect.

To obtain a quantitative estimate of the effect, use is made here of a simple solvable model where the neutron, deuteron, proton, and di-neutron interact through a separable interaction. Such a model has been preferred over a multichannel effective-range approximation. Indeed, the largeness of the size of the di-neutron, and, consequently, that of the range of the interactions involved, restricts seriously the energy region where the effective-range approximation is applicable.[14] On the other hand, in order to adjust the parameters, it is necessary to have a theory whose validity extends over a region much larger than that of the threshold effect.

1. THE INTERACTION

The properties of the neutron, proton, and deuteron are well known. The state of the di-neutron, if it exists, is a $^1S_0$ state to be considered, according to an approximate charge independence, analogous to the virtual state of the deuteron. Therefore, it must be considered as a spin-0 particle. Its binding energy is presumably small. Various authors[3,6,9] have obtained upper bounds for this energy. None of these estimates allows the definite rejection of a binding energy between zero and a few hundreds of keV.

The processes to be considered are the n-d elastic scattering and the $n+d \leftrightarrow p+n_2$ reaction. Besides the $n+d$ and $p+n_2$ channels, the three-body channel is present. We describe such a situation by considering wave functions depending on a channel index and by introducing an interaction which is a Hermitian matrix in the two-body channels. In so doing we neglect, in the effective interactions among the two-body channels, the energy dependence due to the presence of the three-body channel.

The threshold effect in the n-d cross section is due to the S wave of the channel $p+n_2$. This is a pure $^2S_{1/2}$

† Some preliminary results of the present investigation have been communicated at the International Symposium on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962.

[1] M. Y. Colby and R. N. Little, Phys. Rev. **70**, 437 (1946).
[2] D. N. Kundu and M. L. Pool, Phys. Rev. **73**, 22 (1948).
[3] N. Feather, Nature **162**, 213 (1948).
[4] Los Alamos Scientific Laboratory, Phys. Rev. **79**, 238 (1950).
[5] F. W. Fenning and F. R. Holt, Nature **165**, 722 (1950).
[6] K. M. Watson and R. N. Stuart, Phys. Rev. **82**, 738 (1951).
[7] A. J. Ferguson and J. H. Montague, Phys. Rev. **87**, 215 (1952).
[8] B. L. Cohen and T. H. Handley, Phys. Rev. **92**, 101 (1953).
[9] R. Philips and K. Crowe, Phys. Rev. **96**, 484 (1954).
[10] V. V. Komarov and A. M. Popova, Izvest. Akad. Nauk SSSR Ser. Fiz. **24**, 1153 (1960).
[11] K. Ilakovac, L. G. Kuo, M. Petravić, and I. Šlaus, Phys. Rev. **124**, 1923 (1961).
[12] See, for example, L. Fonda, Suppl. Nuovo Cimento **20**, 116 (1961).
[13] A. I. Baz and Ya. A. Smorodinski, Compt. Rend. du Congr. Intern. Phys. Nucl., Paris, 1958, p. 579.

[14] Taking, for example, 50 keV for the binding energy of the di-neutron, a simple calculation gives for the width ΔE of this region $\Delta E \sim 1/2\mu r_0^2 \approx 40$ keV, where μ is the reduced mass and r_0 is the largest of the ranges of the interactions.

state. Let us consider the sector of Hilbert space of our system containing the $^2S_{1/2}$ state of the $p+n_2$ channel. The conservation of the total angular momentum restricts our consideration to the $^2S_{1/2}$, $^2P_{1/2}$, $^4P_{1/2}$, and $^4D_{1/2}$ states of the $n+d$ channel and to the $^2S_{1/2}$ and $^2P_{1/2}$ states of the $p+n_2$ channel. The conservation of parity rules out P waves. Therefore, only the $^2S_{1/2}$ and $^4D_{1/2}$ states of the $n+d$ channel and the $^2S_{1/2}$ state of the $p+n_2$ channel are to be considered. As we deal with rather low energies the D wave is not appreciably coupled with S wave, and it can be neglected. In this sector there exists a bound state: the triton. As only S waves are present in the sector of interest, the chosen interaction will give only S-wave scattering besides having the triton as a bound state.

In order to get a soluble model we use a separable interaction. The Hamiltonian, therefore, reads:

$$H=H_0+H_I,$$

$$H_0=\sum_{i=1}^{2}\int d^3p[(p^2/2\mu_i)+E_i]|i\mathbf{p}\rangle\langle i\mathbf{p}|\,, \qquad (1.1)$$

$$H_I=-\sum_{i,j=1}^{2}\int\int d^3pd^3p'\,|i\mathbf{p}\rangle f_i(p)f_j(p')\langle j\mathbf{p}'|\,,$$

where the values 1 and 2 of the indices indicate, respectively, the $n+d$ and $p+n_2$ channels and $|i\mathbf{p}\rangle$ represents the state of the i pair with relative momentum $\mathbf{p}$ which satisfies

$$\langle i\mathbf{p}|j\mathbf{p}'\rangle=\delta_{ij}\delta(\mathbf{p}-\mathbf{p}')\,.$$

μ_i is the reduced mass in the i channel. E_i is the energy of the ith threshold. The requirements of Hermiticity and time invariance assure us that the various $f_i(p)$ are real. These functions depend only on p owing to the requirement of rotational invariance of the theory.

We will make the following choice for the functions $f_i(p)$:

$$f_i(p)=\alpha_i/p^2+\beta_i^2. \qquad (1.2)$$

The four real quantities α_i, β_i are the parameters of the model.

2. BOUND STATE AND SCATTERING STATES

A. The Bound State (Triton)

The equation for the bound state is

$$H|t\rangle=E_0|t\rangle,$$

with normalizable $|t\rangle$. We put

$$|t\rangle=\sum_{i=1}^{2}\int d^3p\; c_i(\mathbf{p})|i\mathbf{p}\rangle$$

and we obtain for $c_i(\mathbf{p})$ the two following coupled equations

$$c_i(\mathbf{p})[(p^2/2\mu_i)+E_i-E_0]=f_i(p)\left[\sum_{j=1}^{2}\int d^3p'c_j(\mathbf{p}')f_j(p')\right],$$

which allow the only solution

$$c_i(\mathbf{p})=Nf_i(p)/[(p^2/2\mu_i)+E_i-E_0]$$

(where N is a normalization factor), if E_0 satisfies the equation

$$\sum_{j=1}^{2}\int d^3p\frac{f_j^2(p)}{[(p^2/2\mu_j)+E_j-E_0]}=1\,. \qquad (2.1)$$

Equation (2.1) admits either one or no solutions according to

$$\sum_{j=1}^{2}\int d^3p\; f_j^2(p)/[(p^2/2\mu_j)+E_j]$$

being, respectively, more or less than one. With our choice of $f_i(p)$, Eq. (2.1) becomes

$$\sum_{j=1}^{2} 2\mu_j\alpha_j^2\pi^2/\beta_j\{\beta_j+[2\mu_j(E_j-E_0)]^{1/2}\}^2=1\,. \qquad (2.2)$$

B. Scattering States

The T matrix is

$$T_{ji}(\mathbf{p}',\mathbf{p})=\langle j\mathbf{p}'|H_I+H_I\frac{1}{E^{(+)}-H}H_I|i\mathbf{p}\rangle.$$

Simple calculations lead to

$$T_{ji}(\mathbf{p}',\mathbf{p})=-f_j(p')f_i(p)\left[1-\sum_{l,m=1}^{2}\int\int d^3p''d^3p''' \times f_l(p'')f_m(p''')\langle m,\mathbf{p}'''|\frac{1}{E^{(+)}-H}|l,\mathbf{p}''\rangle\right].$$

If we put

$$G_{ml}(\mathbf{p}',\mathbf{p})=\langle m,\mathbf{p}'|\frac{1}{E^{(+)}-\mathrm{H}}|l,\mathbf{p}\rangle$$

$$G_{0ml}(\mathbf{p}',\mathbf{p})=\langle m,\mathbf{p}'|\frac{1}{E^{(+)}-H_0}|l,\mathbf{p}\rangle,$$

the T matrix can be written as

$$T_{ji}(\mathbf{p}',\mathbf{p})=-f_j(p')f_i(p)[1+\mathrm{Tr}(H_IG)].$$

As the relation

$$[1+\mathrm{Tr}(H_IG)][1-\mathrm{Tr}(H_IG_0)]=1$$

holds, we finally obtain

$$T_{ji}(\mathbf{p}',\mathbf{p})=-\frac{f_j(p')f_i(p)}{1-\mathrm{Tr}\,(H_IG_0)} =-\frac{f_j(p')f_i(p)}{1+\sum_{l=1}^{2}\int\frac{d^3pf_l^2(p)}{E^{(+)}-E_l-(p^2/2\mu_l)}}\,. \qquad (2.3)$$

As requested, the scattering is isotropic. Assuming for $f_i(p)$ Eqs. (1.2), the integral in the denominator can be calculated as follows:

$$\int \frac{d^3p f_l^2(p)}{E^{(+)}-E_l-(p^2/2\mu_l)} = -\frac{2\pi^2\mu_l\alpha_l^2}{\beta_l\{\beta_l-i[2\mu_l(E-E_l)]^{1/2}\}^2} \quad \text{if} \quad E>E_l,$$

$$\int \frac{d^3p f_l^2(p)}{E^{(+)}-E_l-(p^2/2\mu_l)} = -\frac{2\pi^2\mu_l\alpha_l^2}{\beta_l\{\beta_l+[2\mu_l(E_l-E)]^{1/2}\}^2} \quad \text{if} \quad E<E_l.$$

Putting these expressions in (2.3), we obtain

$$T_{11}(E) = -\alpha_1^2[\beta_1^2+2\mu_1(E-E_1)]^{-2} \times\left[1-\frac{2\pi^2\mu_1\alpha_1^2}{\beta_1\{\beta_1-i[2\mu_1(E-E_1)]^{1/2}\}^2} - \frac{2\pi^2\mu_2\alpha_2^2}{\beta_2\{\beta_2+[2\mu_2(E_2-E)]^{1/2}\}^2}\right]^{-1} \quad \text{if} \quad E_1<E<E_2, \tag{2.4}$$

and

$$T_{ji}(E) = -\alpha_j\alpha_i[\beta_j^2+2\mu_j(E-E_j)]^{-1}[\beta_i^2+2\mu_i(E-E_i)]^{-1} \times\left[1-\sum_{k=1}^{2}\frac{2\pi^2\mu_k\alpha_k^2}{\beta_k\{\beta_k-i[2\mu_k(E-E_k)]^{1/2}\}^2}\right]^{-1} \quad \text{if} \quad E>E_2. \tag{2.5}$$

The connection between the T matrix and the total scattering cross section is

$$\sigma_{fi}(E) = 64\pi^5\mu_i\mu_f(k_f/k_i)\,|T_{fi}(E)|^2,$$

where

$$E=(k_i/2\mu_i)+E_i.$$

Here and in the following we make use of $\hbar=c=1$ units.

3. THE DETERMINATION OF THE PARAMETERS IN THE INTERACTION

In the formulas of Sec. 2 there appear the quantities E_0, E_i, μ_i, α_i, and β_i. E_0, E_1, μ_1, and μ_2 are given as

$$E_0=-8.492 \text{ MeV},$$
$$E_1=-2.226 \text{ MeV},$$
$$\mu=\mu_1=\mu_2=626 \text{ MeV}.$$

$-E_2$ is the binding energy of the di-neutron. For this energy several values have been considered up to 300 keV. The remaining α_i,β_i are the parameters of the model. Equation (2.2) supplies a first relation among them. Another relation can be obtained by putting the scattering length from the model equal to the experimental doublet scattering length a_d.

As is well known, two sets of scattering lengths are consistent with the experimental data.[15] They are

$$a_d=8.26\times10^{-13} \text{ cm}, \quad a_q=2.6\times10^{-13} \text{ cm}, \tag{3.1a}$$
$$a_d=0.8\times10^{-13} \text{ cm}, \quad a_q=6.38\times10^{-13} \text{ cm}. \tag{3.1b}$$

Let us consider first the value of a_d from set (3.1a). The case a_d from set (3.1b) will be discussed later.

Using Eq. (2.2) and the first set of experimental scattering lengths (3.1a) we get among the four parameters α_i and β_i the following equations:

$$\begin{aligned}
&\frac{2\pi^2\mu}{\beta_1\{\beta_1+[2\mu(E_1-E_0)]^{1/2}\}^2}\alpha_1^2 + \frac{2\pi^2\mu}{\beta_2\{\beta_2+[2\mu(E_2-E_0)]^{1/2}\}^2}\alpha_2^2=1,\\
&\left(\frac{\beta_1 a_d}{2}-1\right)\alpha_1^2 + \frac{\beta_1^4 a_d}{2\beta_2\{\beta_2+[2\mu(E_2-E_1)]^{1/2}\}^2}\alpha_2^2=\frac{\beta_1^4 a_d}{4\pi^2\mu}.
\end{aligned} \tag{3.2}$$

This system can be solved, obtaining α_1^2 and α_2^2 as functions of β_1 and β_2. β_1 and β_2 are, therefore, the two parameters to be still determined.

The quantities $1/\beta_1$ and $1/\beta_2$ have the meaning of ranges of the separable n-d and p-n_2 interactions. We recall that the so-called "radius of the deuteron" is larger than the range of nuclear forces. Since the di-neutron is much less bound than the deuteron, the radius of the di-neutron will be, in turn, larger than that of the deuteron. Owing to these circumstances it is expected that the sizes of these particles play the main role in determining the ranges of their interactions with nucleons. We shall, therefore, limit ourselves to the values of β_1 and β_2 for which $\beta_2<\beta_1$ holds.

By studying the system (3.2) we find that it cannot have positive solutions for α_1^2 and α_2^2 if β_1 is larger than 56 MeV, independently from E_2. On the other hand, if $\beta_2<\beta_1<56$ MeV the solutions are both positive.

Only few data on the n-d doublet scattering are available besides the scattering length. An accurate phase-shift analysis of the n-d scattering based on the choice of set (3.1a) is given by Adair *et al.*[16] From these phase shifts the n-d doublet scattering cross section can be obtained.

With the aid of an electronic computer (IBM 1620 of the Centro di Calcolo dell'Università di Trieste) the total cross section from the model for the elastic process has been calculated for a large number of values of β_1 and β_2 in the ranges indicated above for each chosen value of E_2.

[15] See, for example, M. Verde, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 170; for recent references see L. Spruch and L. Rosenberg, Phys. Rev. **117**, 1095 (1960).

[16] R. K. Adair, A. Okazaki, and M. Walt, Phys. Rev. **89**, 1165 (1953).

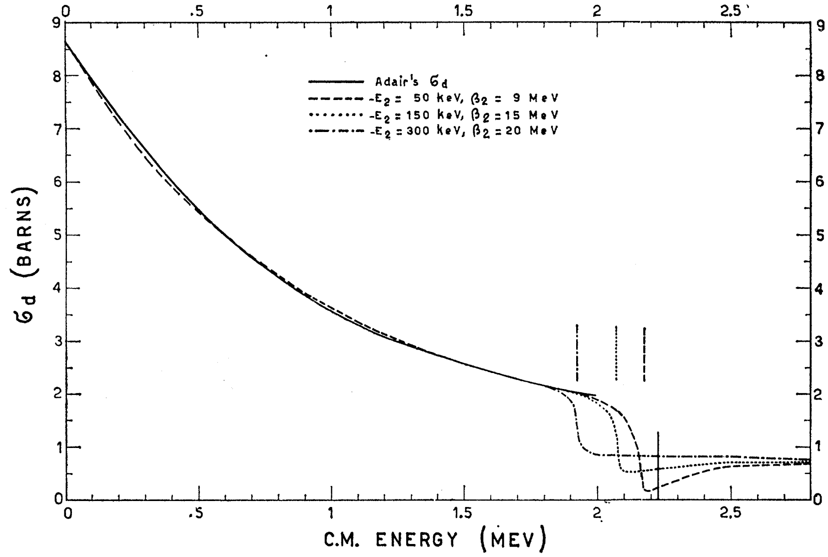

FIG. 1. S-wave doublet n-d scattering cross sections from Adair's phase shifts and from the model ($\beta_1 = 54$ MeV). In the region up to 1.8 MeV the cross sections from the model ($\beta_1 = 54$ MeV) all coincide.

4. ANALYSIS OF THE NUMERICAL RESULTS

Our calculations show that the threshold effect has the shape of a rounded step. This shape is in agreement with the fact that below the considered threshold there is only the elastic channel open and that the tangent of the doublet S-wave phase shift is negative.

We consider the energy range of interest as divided in two regions. The first extends from E_1 to about 500 keV before the threshold energy E_2. The second is the threshold-effect region. The comparison of the cross section from the model with the experimental one can be accomplished only in the first region.

Considering the numerical results one sees that in the first region the cross section is controlled almost exclusively by the value of β_1. The best agreement is obtained for β_1 in the range 53–55 MeV independently from E_2. The agreement is remarkable as it is shown in Fig. 1 which refers to the value $\beta_1 = 54$ MeV. For values of β_1 smaller than 53 MeV the agreement rapidly worsens.

Owing to the lack of sensitivity of the cross section on β_2 in the first region, the comparison with Adair's data leaves β_2 quite undetermined. However, on the basis of the consideration made in Sec. 3, values of β_2 near the value chosen for β_1 can be ruled out. In fact, the di-neutron, owing to its very small binding energy is, if it exists, appreciably larger than the deuteron. That consideration is supported by the fact that the value of β_1 obtained from the comparison with Adair's data corresponds to a range of about 4 F for the n-d interaction, a value near the "radius of the deuteron."[17]

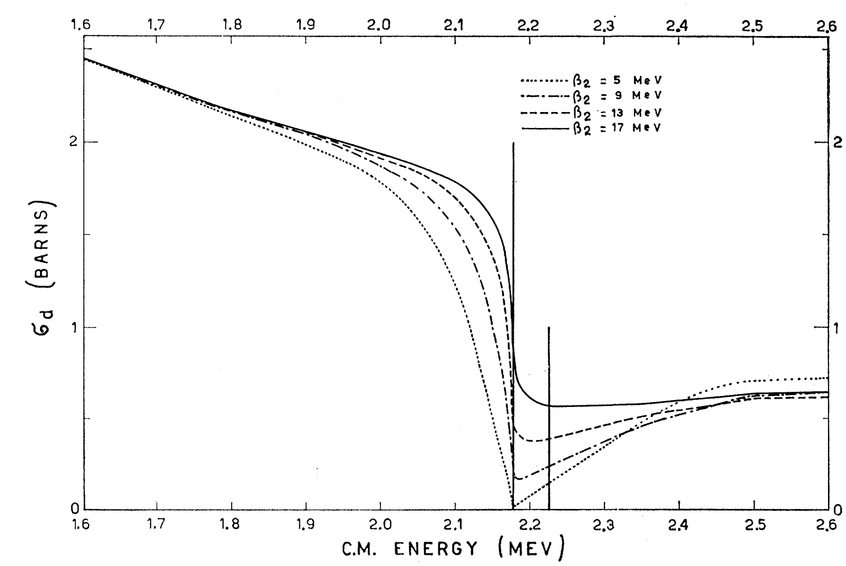

FIG. 2. Threshold effect region of the calculated S-wave doublet n-d scattering cross sections. $\beta_1 = 54$ MeV, dineutron binding energy = 50 keV.

[17] See, for example, R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955).

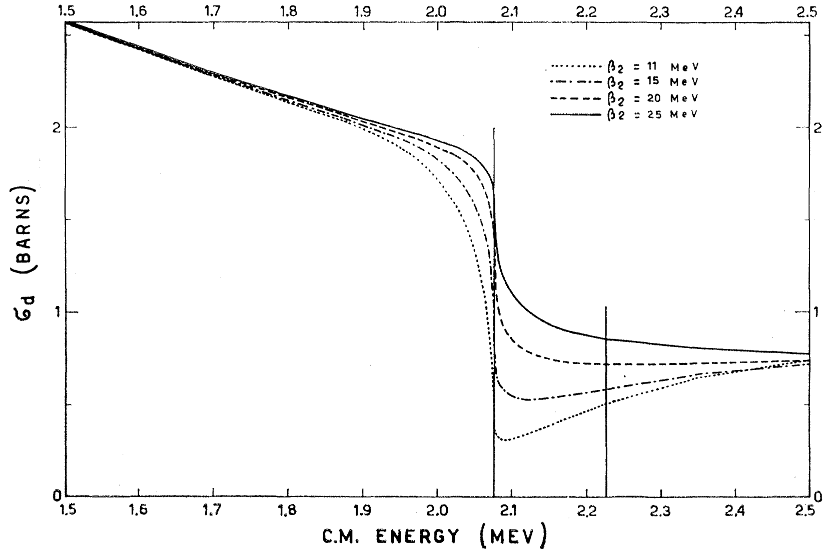

FIG. 3. Threshold effect region of the calculated S-wave doublet n-d scattering cross sections. $\beta_1 = 54$ MeV, di-neutron binding energy = 150 keV.

A rough estimate of β_2 may be obtained by assuming that the ratio β_2/β_1 approximately equals $(E_2/E_1)^{1/2}$. This estimate, taking $\beta_1 = 54$ MeV, would give for β_2 the values 8, 14, 20 MeV corresponding, respectively, to the values 50, 150, 300 keV for the binding energy of the di-neutron.

In Figs. 2–4 the doublet cross section, in the threshold effect region, is plotted for the three above-mentioned values of the binding energy for $\beta_1 = 54$ MeV and various values of β_2 in rather wide ranges about the values indicated above. As it is seen, at fixed binding energy the threshold effect changes both in shape and in magnitude with β_2. The shape of the effect is a rounded step followed by a minimum for small values of β_2. The minimum rises and disappears for increasing β_2. The height of the effect decreases for increasing β_2, but always remains rather large. When the binding energy varies, being β_2 fixed, the effect obviously shifts but it remains almost unchanged. However, by varying the binding energy, the corresponding set of values of β_2 will change according to the recipe given above and as a consequence the effect will be modified.

In Fig. 5 the reaction cross sections are plotted for the same values of the parameters as in Fig. 1. In Table I, the heights and the widths of the steps for all diagrams of Figs. 2–4 are listed.

If $\beta_1 = 53$ or 55 MeV, the height of the effect is not significantly different from the height when $\beta_1 = 54$ MeV. On the contrary, the width of the threshold effect region tends to decrease for β_1 increasing from 53 to 55 MeV. As to the dependence on β_2, our calculations show that if β_2 is increased up to values near β_1 the effect appreciably decreases, but these values are ruled out by the previous considerations.

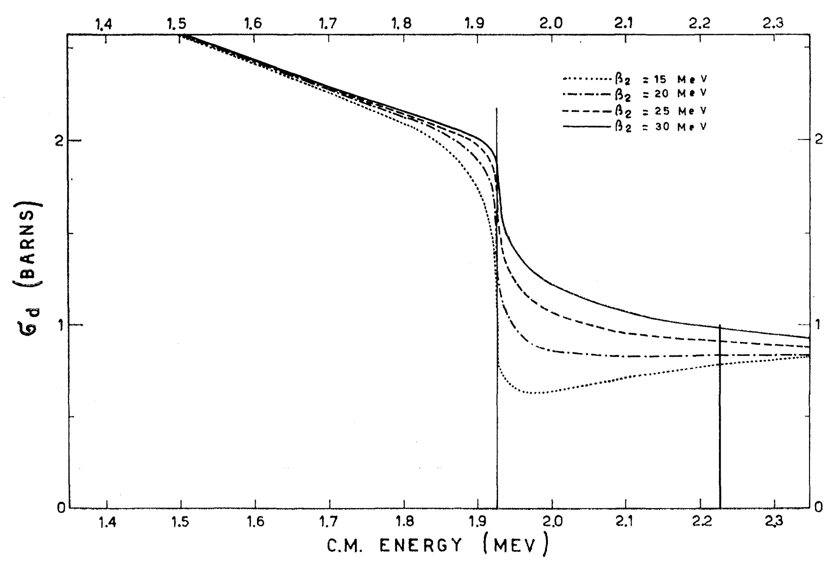

FIG. 4. Threshold effect region of the calculated S-wave doublet n-d scattering cross sections. $\beta_1 = 54$ MeV, di-neutron binding energy = 300 keV.

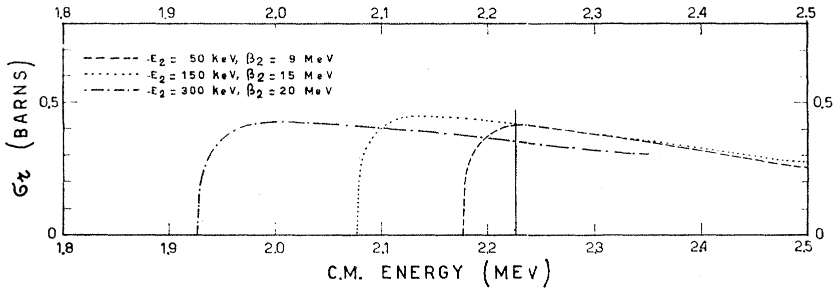

FIG. 5. S-wave $n+d \rightarrow p+n_2$ reaction cross sections from the model ($\beta_1=54$ MeV).

5. FINAL RESULTS AND CONCLUSIONS

The total elastic cross section is the following sum of the doublet and quartet cross sections:

$$\sigma_{\text{tot}}=\tfrac{1}{3}\sigma_d+\tfrac{2}{3}\sigma_q.$$

Taking into account the values of σ_{tot} at the various considered threshold energies[18] and the absolute heights of the steps in σ_d listed in Table I, one can obtain the percent heights of the effect in σ_{tot}. These, too, are listed in Table I. As seen, the height of the effect is contained between 15 and 23% of σ_{tot} in all cases. These values, with those of the widths, are such as not to make hopeless the attempt to detect experimentally the effect if the di-neutron exists.

Considering the scattering length from set (3.1b) we note that in this case the doublet contribution to σ_{tot} is much smaller than before. As some calculations we performed in this case show that the effect in σ_d is of equal or smaller order than that given by using (3.1a), the step is practically not perceptible in σ_{tot}.

TABLE I. Values of α_1^2, α_2^2, absolute heights h_d in σ_d, percent heights h_{tot} in σ_{tot}, and widths w of the effect for $\beta_1=54$ MeV and for the same values of the di-neutron binding energy and of the parameter β_2 of Figs. 2–4. The quantity w is evaluated as the width of the energy region from the point where the total cross section deviates more than 3% from that in absence of the effect, up to the three-body production threshold.

$-E_2$ (keV)	β_2 (MeV)	α_1^2 (MeV²)	α_2^2 (MeV²)	h_d (b)	h_{tot} (%)	w (keV)
50	5	83.4	0.289	1.5	23	180
	9	82.9	0.607	1.4	22	140
	13	82.4	1.015	1.3	20	100
	17	82.0	1.524	1.2	19	70
150	11	82.9	0.761	1.4	22	230
	15	82.4	1.200	1.2	18	190
	20	81.9	1.895	1.1	17	160
	25	81.3	2.777	1.0	15	150
300	15	82.8	1.116	1.3	19	340
	20	82.2	1.770	1.1	16	320
	25	81.7	2.603	1.0	15	300
	30	81.1	3.639	1.0	15	300

[18] *Neutron Cross Sections*, compiled by D. J. Hughes and R. B. Schwartz (McGraw-Hill Book Company, Inc., New York, 1955).

Therefore the detection of the threshold effect would establish the existence of the di-neutron and would support the choice of set (3.1a) of scattering lengths. This conclusion is independent from the reliability of our model.

On the contrary, if an effect of the same order of magnitude as the one calculated, should be excluded experimentally, two cases would be possible:

(1) The di-neutron does not exist; then, the model does not supply any suggestion about the scattering lengths.

(2) The di-neutron does exist; then, set (3.1b) is favored according to the model.

The model we have used assumes a separable interaction and does not take into account the energy dependence coming from the presence of the three-body continuum states. The ticklish point about it is that it is hard to know how bad these approximations are. We note, however, that, in spite of this, we have succeeded, using only three of the four parameters, in reproducing a large set of experimental data remarkably well. In fact, besides the binding energy of triton and the doublet-scattering length it is possible to fit, quite accurately, the cross section up to energies much beyond the range of validity of the effective-range approximation. Moreover, the value found for β_1 well agrees with the size of the deuteron. This fact, which is encouraging by itself, enables us to fix approximately the value of β_2 according to the size of the di-neutron.

ACKNOWLEDGMENTS

We thank Professor L. Fonda for stimulating discussions. One of us (R. A.) is greatly indebted to Professor R. Ricamo for his kind hospitality at the Istituto di Fisica dell'Università di Catania in June, 1962 and to Professor A. Agodi for discussions. Two of us (G. C. G. and A. R.) are very thankful to professor P. Budini for his kind hospitality at the Istituto di Fisica dell'Università di Trieste in July–August, 1962, where a part of this work was carried out.